UGC MINOR RESEARCH PROJECT

ON VARIOUS CORDIAL LABELING TECHNIQUES

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ज्ञान-विज्ञान विमुक्तये

Undertaken By
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FINAL REPORT OF THE WORK DONE ON THE MINOR RESEARCH PROJECT

1. Title of the Project:: On Various Cordial Labeling Techniques
2. Name and address of the Principal Investigator: Dr. Udayan M. Prajapati
3. Name and Address of the institution: Department of Mathematics. St. Xavier's College (Autonomous), Ahmedabad, Gujarat.
4. UGC Approval Letter No. and Date:No. F. 47-903/14-(WRO) Dt.15.3.2015
5. Date of Implementation: 15-3- 2013.
6. Tenure of the Project: Two Years (March 2015 to March 2017)
7. Total Grant Allocated: Rs. 180,000/-
8. Total Grant Received:Rs.135, 000/-
9. Final Expenditure: Rs. 103419/-
10. Title of the Project: On Various Cordial Labeling Techniques.
11. Objectives of the Report:-
    - To study various cordial labeling techniques in various graphs.
    - To study cordiality of various graphs obtained by various graph operations.
    - To define various graphs in the context of cordiality of several types.
12. Whether objectives were achieved: Yes.
13. Achievements from the Project : Seven papers have been published in various peer reviewed/UGC recognize journals
14. Summary of the publications: Copy attached.
15. Whether any Ph.D. enrolled/produced out of the project : some of the research scholars have done joint work with me
16. No. of publications out of the project:-Seven (attached)

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Chapter 1

Cordial Labeling

1.1: Introduction

Graph Theory has a wide application in the field of Mathematics, Network Analysis, Operations Research, Management Science, Computer Science and Mobile Application. Graph labeling is one of the important concepts of Graph Theory which was developed on the basis of Graph Colouring.

Here Graph means simple, finite, undirected and non-trivial graph $G = (V, E)$ with the vertex set V and the edge set E. The number of elements of V, denoted as $|V|$ is called the order of the graph G while the number of elements of E, denoted as $|E|$ is called the size of the graph G. For various graph theoretic notations and terminology we follow Gross and Yellen [1] whereas for number theory we follow D. M. Burton [2]. We will give brief summary of definitions and other information which are useful for the present investigations.

Definition 1.2: If the vertices of the graph are assigned values subject to certain conditions then it is known as **graph labeling**.

For latest survey on graph labeling we refer to J. A. Gallian [3]. Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in last four decades. For any graph labeling problem following three features are really noteworthy:

- a set of numbers from which vertex labels are chosen;
- a rule that assigns a value to each edge;
- a condition that these values must satisfy.

The present work is aimed to discuss several various types of graph labellings.

Definition 1.3: A function $f : V(G) \to \{0, 1\}$ is called a **binary vertex labeling** of G and $f(v)$ is called the label of the vertex v of G under f.

If for an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0, 1\}$ is given by $f^*(e) = |f(u) - f(v)|$, $v_f(i)$ = number of vertices of G having label i under f and $e_f(i)$ = number of edges of G having label i under f^* for where $i = 0$ or 1.

Definition 1.4: A binary vertex labeling f of a graph G is called a **cordial labeling** if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is **cordial** if it admits cordial labeling.

1.5: Existing results of cordial labeling

Tree, $K_{m,n}$, K_n (iff $n \leq 3$), fans, W_n (iff $n \not\equiv 3(mod\ 4)$), unicyclic graph (other than C_{4k+2}), multiple shells, t-ply graph $P_t(u,v)$ (except when it is Eulerian and the number of edges $e \equiv 2(mod\ 4)$), helms, closed helms, generalized helms are cordial, the graph obtained by joining two copies of cycle C_n by a path of arbitrary length, the graph obtained by joining two copies of cycle C_n with twin chords by a path of arbitrary length (where chords form two triangles and one cycle C_{n-2}), star of wheel W_n, star of Petersen graph, the graph obtained by joining two copies of wheel by a path of arbitrary length, Cycle with one chord, the graph obtained by joining two copies of cycle C_n with one chord by a path of arbitrary length, Barycentric subdivision of cycle with twin chords, Barycentric subdivision of cycle C_n with triangle (except $n = 0, 2(mod\ \ 4)$), $C_n(C_n)$ (except $n = 2(mod\ \ 4)$), the path union of k copies of $C_n(C_n)$, the graph G obtained by joining two copies of $C_n(C_n)$ by a path of arbitrary length, shadow graph $D_2(C_n)$ of cycle C_n, the path union of k copies of shadow graph $D_2(C_n)$ of cycle C_n, the graph obtained by joining two copies of shadow graph $D_2(C_n)$ by a path of arbitrary length, arbitrary supersubdivision of path P_n, $S(M_n)$, $S(F_n)$ (for $n \geq 2$), $S(K_{m,n})$ are cordial.

1.6: The following results have been derived.

1. The graph obtained by duplicating all the vertices of the flower graph Fl_n is cordial.
2. The graph obtained by duplicating all the rim vertices of the flower Fl_n is cordial.
3. The graph obtained by duplicating all the vertices of a lotus inside circle Lc_n is cordial.
4. The graph obtained by duplicating all the vertices except the apex vertex of the sunflower graph $V[n,s,t]$ is cordial.
5. The graph obtained by duplicating all the vertices of the sunflower graph $V[n,s,t]$ is cordial.
6. The graph obtained by duplicating all the vertices of the sunflower graph SFL_n is cordial.
7. The graph obtained by duplicating all the rim vertices of the sunflower graph SFL_n is cordial.
8. The graph obtained by duplicating all the vertices of the flower snark graph J_n is cordial.
9. The graph obtained by duplicating all the vertices of the web Wb_n is cordial.
10. The graph obtained by duplicating all the pendent vertices of the web Wb_n is cordial.
11. The graph obtained by duplicating the outer rim vertices and the apex of the web Wb_n is cordial.
12. The graph obtained by duplicating all the vertices except the apex vertex of the web Wb_n is cordial.
13. The graph obtained by duplicating all the inner rim vertices and the apex vertex of the web Wb_n is cordial.
14. The graph obtained by duplicating all the edges other than spoke edges of the web Wb_n is cordial.
15. The graph obtained by duplicating all the vertices of the armed helm AH_n is cordial.
16. The graph obtained by duplicating all the vertices other than the rim vertices of the armed helm AH_n is cordial.
17. The graph obtained by duplicating all the rim vertices of the armed helm AH_n is cordial.
18. The graph obtained by duplicating all the vertices except the apex vertex of the armed helm AH_n is cordial.

19. The graph obtained by duplicating all the edges other than spoke edges of of the armed helm AH_n is cordial.

20. The graph obtained by duplication of every vertex by an edge in C_n is cordial.

21. The graph obtained by duplication of every edge by a vertex in C_n is cordial if $n \not\equiv 2 \pmod 4$.

22. The graph obtained by duplication of every edge by a vertex in C_n is not cordial if $n \equiv 2 \pmod 4$.

23. The graph obtained by duplication of every vertex by an edge in P_n is cordial.

24. The graph obtained by duplication of every edge by a vertex in P_n is cordial if $n \not\equiv 3 \pmod 4$.

25. The graph obtained by duplication of every edge by a vertex in P_n is not cordial if $n \equiv 3 \pmod 4$.

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Chapter 2

Edge Product Cordial Labeling

Definition 2.1:

For a graph $G = (V(G), E(G))$ having no isolated vertex a function $f : E(G) \rightarrow \{0, 1\}$ is called an **edge product cordial labeling** of G, if the induced vertex labeling function defined by the product of labels of incident edges to each vertex be such that the number of edges with label 0 and the number of edges with label 1 differ by at most 1 and the number of vertices with label 0 and the number of vertices with label 1 also differ by at most 1.

2.2: Existing results of cordial labeling

The concept of edge product cordial labeling was introduce by Vaidya and Barasara [21] in which they proved that C_n for n odd, trees with order greater than 2, unicyclic graphs of odd order, crowns, armed crowns, helms, closed helms, webs, flowers graph are edge product cordial.They also proved that wheel and gear for even are not edge product cordial. They also [22] proved that T_n, DT_n for odd, Q_n for odd, DQ_n for odd are edge product cordial labeling.They also proved that DT_n for even, Q_n for even, DQ_n for even, DF_n are not edge product cordial labeling.

2.3: The following results have been derived.

1. The graph obtained by duplication of an arbitrary vertex of the cycle in a crown graph is an edge product cordial graph.
2. The graph obtained by duplication of an arbitrary vertex of the cycle by a new edge in a crown graph is edge product cordial graph.
3. The graph obtained by duplication of an arbitrary edge by a new vertex in a crown graph is edge product cordial.
4. The graph obtained by duplication of each pendent vertex by a new vertex in a crown graph is edge product cordial graph.
5. The graph obtained by duplication of each of the vertices of degree three by an edges in a gear graph is edge product cordial graph.
6. The graph obtained by duplication of each of the pendent vertices by a new vertex in a helm graph is edge product cordial graph.

7. Closed web graph CWb_n is not an edge product cordial graph.

8. Lotus inside circle LC_n is not an edge product cordial graph.

9. Sunflower graph SF_n is edge product cordial graph for $n \geq 3$.

10. The graph obtained by duplication of each of the vertices w_i for $i = 1, 2 \ldots, n$ by new vertex in sunflower graph SF_n is an edge product cordial graph if and only if n is even.

11. The graph obtained from duplication of each of the vertices w_i for $i = 1, 2, \ldots, n$ by a new edges f_i in sunflower graph SF_n is an edge product cordial graph.

12. The graph obtained by duplication of each of the vertices in sunflower graph SF_n is not an edge product cordial graph.

13. The graph obtained by subdividing the edges w_iv_i and $w_iv_{i+1} \pmod{n}$ for all $i = 1, 2, \ldots, n$ by a vertex in sunflower graph SF_n is an edge product cordial.

14. The graph obtained by flower graph Fl_n by adding n pendant vertices to apex vertex v_0 is edge product cordial graph.

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Chapter 3

Product Cordial Labeling

Definition 3.1: A binary vertex labeling of a graph G with induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ defined by $f^*(e = uv) = f(u)f(v)$ is called a **product cordial labeling** if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph is called product cordial if it admits a product cordial labeling.

3.2: Existing results of product cordial labeling:

The notion product cordial labeling was introduced by Sundaram, Ponraj and Somasundaram [29]. They proved that following graphs are product cordial graph: tree, unicyclic graph of odd order, triangular snakes, dragons and helms. Vaidya and Barasara [30] also proved few results on product cordial graph. They showed that following graphs are product cordial: gear graph obtained from wheel graph W_n if and only if n is odd, web graph, flower graph and closed helm.

The graphs obtained by joining apex vertices of k copies of stars, shells and wheels to a new vertex, shadow graph of cycle C_n, the cycle with one chord, the cycle with twin chords, the friendship graph and the middle graph of path, the graphs obtained by duplication of one edge, mutual vertex duplication and mutual edge duplication in cycle are product cordial.

3.3: The following results have been derived.

1. The graph obtained by duplication of each vertex of degree two in the gear graph is not a product cordial graph.
2. The graph obtained by duplication of each vertex of degree two by an edge in the gear graph is a product cordial graph.
3. The graph obtained by duplication of each vertex of degree three in the gear graph is not a product cordial graph.
4. The graph obtained by duplication of each vertex of degree three by an edge in the gear graph is a product cordial graph.
5. The graph obtained by duplication of the apex vertex in the gear graph is not a product cordial graph.
6. The graph obtained by duplication of the apex vertex by an edge in the gear graph is not a product cordial graph if n is odd.
7. The graph obtained by duplication of the apex vertex in the gear graph by an edge admits product cordial labeling if n is even.
8. The graph obtained by switching of a vertex of degree two in gear graph is a product cordial graph.

9. The graph obtained by applying vertex switching on a single vertex of degree three in gear graph is a product cordial graph.
10. The graph obtained by applying vertex switching on the apex vertex in gear graph is a product cordial graph if n is odd and it is not product cordial graph if n is even.

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Chapter 4

Prime Cordial Labeling

Definition 4.1: A **prime labeling** of a graph G of order n is an injective function $f : V \to \{1, 2, \ldots, |V|\}$ such that for every pair of adjacent vertices u and v, $gcd(f(u), f(v)) = 1$. The graph which admits prime labeling is called a **prime graph**.

4.2: Existing results of prime labeling:

The notion of prime labeling was originated by Entringer and was discussed in A.Tout. Many researchers have studied prime graphs. It has been proved by H. L. Fu and C. K. Huang [7] that P_n is a prime graph. It has been proved by S. M. Lee [8] that wheel graph W_n is a prime graph if and only if n is even. T. Deretsky [9] has proved that cycle C_n is a prime graph.

Definition 4.3: A bijection function $f : V \to \{1, 2, \ldots, |V|\}$ is called a **prime cordial labeling** of a graph G if the corresponding induced edge labeling $f^* : E(G) \to \{0, 1\}$ given by $f^*(e) = 1$ if $\gcd(f(u), f(v)) = 1$ and $f^*(e) = 0$ if $\gcd(f(u), f(v)) > 1$ for every edge $e = uv$ with $|e_f(0) - e_f(1)| \leq 1$. A graph which admits a prime cordial labeling is said to be a **prime cordial graph**.

4.4: Existing results of prime cordial labeling:

C_n, P_n, $K_{1,n}$, cycle C_n with (one chord, twin chords or a triangle), P_n if and only if $n \neq 3$ or 5, $K_{1,n}$ (n odd), the graph obtained by subdividing each edge of $K_{1,n}$ (iff $n \geq 3$), bistars, dragons, crowns, ladders, $K_{2,n}$ (if n is even and if there exists a prime p such that $3p < n+2 < 4p$, $K_{3,n}$ if n is odd and if there exists a prime p such that $5p < n+3 < 6p$, sun graphs $C_n \odot K_1$, C_n with a path of length $n-3$ attached to a vertex, the total graph of P_n, the total graph of C_n for $n \geq 5$, $P_2[P_m]$ for all $m \geq 5$, the graph obtained by joining two copies of a fixed cycle by a path, the graph obtained by switching of a vertex of C_n except for $n = 5$, the friendship graph F_n for $n \geq 3$, P_n^2 (for $n = 6$ and $n \geq 8$), C_2 (for $n \geq 10$), the shadow graphs of $K_{1,n}$ for $n \geq 4$, the bistar $B_{n,n}$, split graphs of $K_{1,n}$, $B_{n,n}$, the square graph of $B_{n,n}$, the middle graph of P_n (for $n \geq 4$), W_n (iff $n \geq 8$) are prime cordial.

4.5: The following results have been derived.

1. $Y_{2n+1,2}$ is prime cordial for all n.

2. $Y_{2n,2}$ is prime cordial for all $n > 2$.

3. $Y_{n,2}$ is prime cordial for all n except $n = 1, 2$ and 4.

4. $Y_{2n+1,4}$ is prime cordial for all n.

5. $Y_{2n,4}$ is prime cordial for all $n \geq 2$.
6. $Y_{n,4}$ is prime cordial for all $n \geq 3$.
7. $Y_{3,2n}$ is prime cordial.
8. $Y_{3,2n+1}$ is prime cordial.
9. $Y_{3,n}$ is prime cordial for all n grater than 1.
10. $Y_{4,2n}$ is prime cordial for all n grater than 1.
11. $Y_{4,2n+1}$ is prime cordial.
12. $Y_{4,n}$ is prime cordial for all n grater than 2.
13. $Y_{5,n}$ is prime cordial for all n grater than 1.
14. $Y_{6,n}$ is prime cordial for all n grater than 1.
15. $Y_{2p,n}$ is prime cordial for all odd prime p and for all $n > 1$.
16. $Y_{m,n}$ prime cordial for all $m \geq 3$ and $n \geq 2$ except $Y_{4,2}$.

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